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Investigation of the effect of a bend in a transfer line that separates a pulse tube cold head and a pressure wave generator

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Abstract A transfer line between a pulse tube cold head and a pressure wave generator is usually required to isolate the cold head from the vibrations of the compressor. Although it is a common practice to use a thin and narrow straight tube, a bent tube would allow design flexibility and easy mounting of the cold head, such as in a split Stirling type pulse tube cryocooler. In this paper, we report a preliminary investigation on the effect of the bending of the tube on the flow transfer characteristics. A numerical study using commercial computational fluid dynamics model is performed to gain insight into the flow characteristics in the bent tube. Oscillating flow experiments are performed with a straight and a bent tube at a filling pressure of 15 bar and an operating frequency of 40, 50 and 60 Hz. The data and the corresponding numerical simulations point to the hypothesis that the secondary flow in the bent tube causes a decrease in flow at a fixed pressure amplitude.

1. Introduction

For detector cooling, a split-Stirling type pulse tube cryocooler is preferred over other configurations because the cold head is partially isolated from the vibrations of the compressor and it allows flexibility in assembling the cold head to the detector. In the spilt configuration, a narrow tube also called a transfer line, is used to connect the pressure wave generator to the cold head. The additional volume due to the transfer line increases the swept volume of the pressure wave generator, thereby increasing the size of the pressure wave generator (the size is proportional to the swept volume) and reducing the performance of the cooler (increased swept volume increases the acoustic power).

Spoor and Corey [1] investigated the use of a long transfer line to harness the inertance effects of long and thin tube. Although in theory the transfer line approach is promising, in practise the losses in the transfer line is substantial leading to a trade-off between design flexibility and performance. The impedance characteristics of inertance tubes are studied in detail by Lewis et al. [2] and showed that viscous losses cannot be neglected.

Our study will focus on the use of a narrow and a short bent tube as a transfer line. It is observed that a steady flow through a curved tube will induce a secondary flow in the normal plane to the flow [3]. Dev and Ardhapurkar [4] studied the effect of secondary flow on the flow and heat transfer characteristics in a miniature helical coil heat exchanger. The study reports increase in heat transfer rate due to secondary flow but the development of the secondary flow leads to a pressure drop in the tube. Similar flow behaviour is expected with oscillating flow in a tube. Taylor et. al. [5] used two dimensional axisymmetric computational fluid dynamics code to model the pulse tube and predict flow and thermal losses. Since the secondary flow occurs in a plane normal to the main flow direction a three dimensional model is necessary to capture these effects.

The pressure drop in a straight tube is only due to the viscous shear where as in bent or curved tube a transverse pressure gradient develops perpendicular to the plane of flow due to the centrifugal force. The pressure drop in a bent tube is caused by both viscous drag and momentum change resulting from the change in the direction of flow. The later part is quantified in terms of transverse pressure gradient given by Euler’s equation. Hence a curved tube of the same diameter and length of straight tube has more pressure drop. In the case of laminar flow in a straight tube, after the entrance length, the velocity profile is fully developed and is symmetrical about the axis of the flow. However, in case of a bent tube, the velocity profile is first symmetrical in the straight section then velocity profile shifts on one side due to the effect of centrifugal force in the bent section, then again becomes symmetrical about the axis in the straight section. This shift of velocity profile also introduces pressure drop. The shear stress is not considered in Euler’s equation because it considers forces acting in the transverse direction of the flow.

The objective of the present work is to investigate the secondary flow losses induced by a bend under oscillating flow conditions. For this purpose, we will explore numerical methods to capture the oscillating flow characteristics in the tube and experimental methods to characterize the pressure drop and flow in the tube.

2. Secondary flow in a bend

A flow through curved tube induces a pressure gradient in the transverse direction of the flow. This transverse pressure gradient induces a flow, also called secondary flow, in the direction perpendicular to the main flow direction. The intensity of the secondary flow velocity depends on the transverse pressure gradient, as it is the only source that creates this flow. Euler’s equation normal to the streamlines in a flow through a curved tube (see Figure 1), relates the transverse pressure gradient with the geometric parameter, operating conditions and fluid properties.

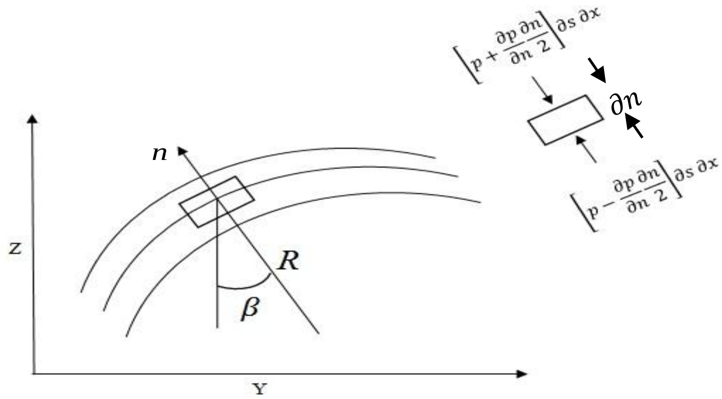

Figure 1. Streamline in the flow domain in the horizontal z-y plane.

In Figure 1, the secondary flow is in the y-z plane and the main flow is in the x direction into the plane of the figure. Consider an element with height ∂n, width ∂s and thickness ∂x, where ∂n is perpendicular to the streamlines, ∂s is elementary distance along the streamline and ∂x is thickness of element into the plane. Applying Newton’s second law in the normal direction to the streamlines.

$$\left(p-\frac{\partial p}{\partial n}\frac{\partial n}{2}\right)\partial s\partial x-\left(p+\frac{\partial p}{\partial n}\frac{\partial n}{2}\right)\partial s\partial x-\left(\rho g\,\mathrm{Cos}\,\beta\right)\partial s\partial n\partial x=\left(\rho a_n\,\mathrm{Cos}\,\beta\right)\partial s\partial n\partial x \quad (1)$$

where p is pressure at a point, β is the angle between the normal and the vertical direction and a_n is the acceleration of fluid particle in the normal direction. The above equation on simplification yields:

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$$-\frac{\partial p}{\partial n} - \rho g \, \mathrm{Cos}\,\beta = \rho a_n \tag{2}$$

Considering terminology in figure 1, $\mathrm{Cos}\,\beta = \frac{\partial z}{\partial n}$

$$\frac{-1}{\rho}\frac{\partial p}{\partial n} - g\frac{\partial z}{\partial n} = a_n\,; \tag{3}$$

The normal acceleration of the fluid element is towards the center of curvature of the streamline in the negative 'n' direction. This equals the centripetal acceleration as shown in equation 4.

$$a_n = \frac{-V^2}{R} \tag{4}$$

In above equation, R is the radius of curvature. Therefore equation 3 reduces to,

$$\frac{1}{\rho}\frac{\partial p}{\partial n} + g\frac{\partial z}{\partial n} = \frac{V^2}{R} \tag{5}$$

For flow in horizontal plane, the gravity term can be neglected hence the equation reduces to

$$\frac{\partial p}{\partial n} = \frac{\rho V^2}{R} \tag{6}$$

Equation 6 shows the relation between the transverse pressure gradient and the design parameter R, operating parameter V and fluid property ρ . The transverse pressure gradient scales with square of axial flow velocity (V). The secondary flow intensity scales proportionally to the transverse pressure gradient. Therefore, secondary flow intensity scales with the square of axial velocity (V). The development of the secondary flow requires energy and it is taken from the flow. This decreases the pressure amplitude of the flow. The pressure drop in a flow through a bend can be studied directly by investigating the secondary flow in the bend.

3. Experimental setup

A test rig is built to characterize the oscillating flow in a transfer line. Figure 2 shows the picture of the set-up. A 250 W electrical power rated pressure wave generator is connected to the test object (a straight or a bent tube). The other end of the tube is connected to a reservoir of volume 104 ml through a valve. The mass flow into the reservoir is controlled by opening the valve. At first, the experiments are performed with a straight copper tube of length 250 mm and an internal diameter of 5 mm. Oscillating pressure is measured at three locations as shown in Figure 2, at the entrance to the tube P_1, exit of the tube P_2 and in the reservoir P_3.

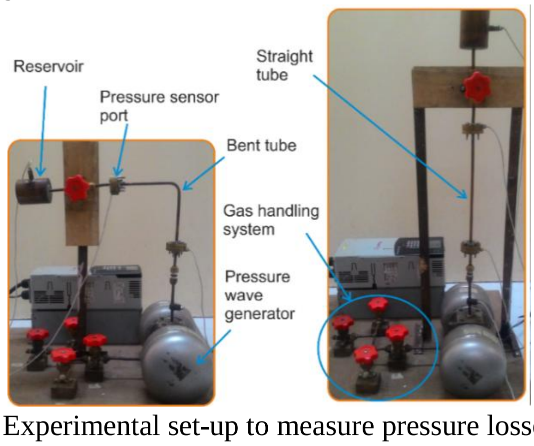

Figure 2. Experimental set-up to measure pressure losses in tubes.

The pressure signal is fed to a lock-in amplifier to extract the pressure amplitude and phase. In all the experiments, helium gas is used, which is fed to the system through the gas handling system. After the

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data acquisition with the straight tube is complete, experiments are carried out with a 90 degrees bent tube. The experimental data will provide information on the macroscopic observable parameters, such as pressure and flow. In order to gain insight into the physics of oscillating flow in the bent tube and to correlate the microscopic effects to the experimental data, we have employed a numerical model for oscillating flow in a tube. The numerical model is a three dimensional Computational Fluid Dynamics (CFD) model using ANSYS CFX. The geometry used in the simulation is shown in Figure 3. Geometric parameters are replica of the actual experimental set-up. The main boundary and solver parameters are summarized in Table 1. The inlet of the connecting tube is specified as 'opening' with a pressure wave as the boundary condition. The 'opening' boundary condition permits to and fro flow at the boundary surface. The fluid domain is helium gas and is considered isothermal at a temperature of 300 K. Mesh independent analysis is performed to validate the dependency of numerical solution on the grid density. A number of transient simulations are performed with several operating parameters i.e., pressure ratio at inlet and charge pressure to gain insights into the development of secondary flow in the bend.

Table 1. Boundary conditions and solver parameters.

Boundary and Solver parameters	Condition
Solution type	Transient
Total time (s)	Depends on frequency (for 5 complete cycles)
Time steps (s)	20 division per cycle
Frequency (Hz)	40
Inlet (Opening)	Pressure wave
Reservoir end	Wall
Turbulence Model	K-epsilon
Advection Scheme	High Resolution
Transient Scheme	Second Order Backward Euler

Referring to section 2, a flow through a curvature results in the development of a secondary flow. Figure 3 shows a typical result of the numerical simulation at the cross-section of the bend. The filling pressure in this specific case is 15 bar, operating frequency is 40 Hz and the oscillating pressure amplitude is 0.127 MPa.

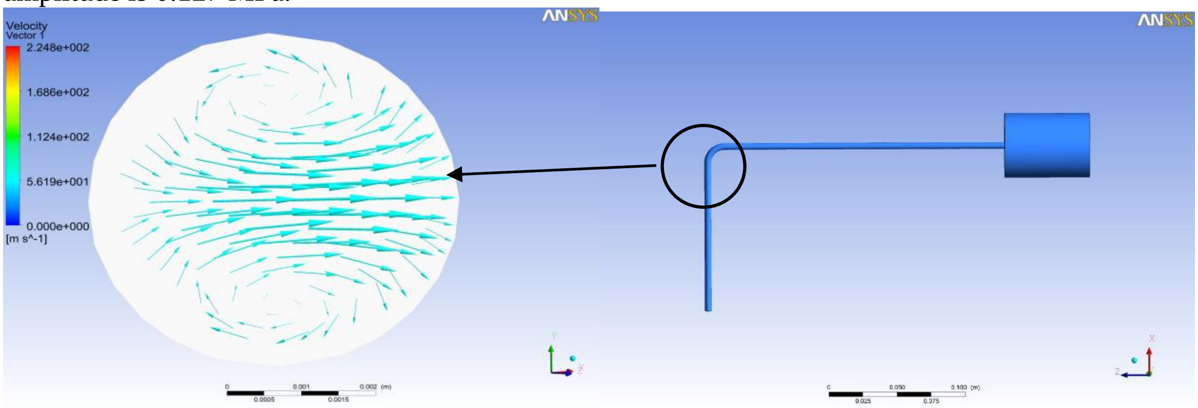

Figure 3. Visualisation of secondary flow in the cross-section of the tube.

In figure 3, two counter rotating eddies can be visualized in the bent section, which is similar to the flow pattern observed by Dean [1] for a steady flow through a curved tube. The difference is that the direction of the velocity vectors changes in the oscillating flow. The arrow heads are the velocity vectors that show the direction and magnitude of the velocity. Magnitude of the velocity is higher at the center of the tube and reduces as the flow progresses to the outer end of the tube. This is attributed to the surface effects at the boundary (shear stress and velocity gradient in the boundary layer). Figure 3 shows secondary flow at a time step. In this case the velocity of secondary flow varies from 0 to 224 m/s on the plane of secondary flow. The intensity of secondary flow also varies with time, because of oscillating flow. Due to the variation of volume flow in the bend, the velocity of flow through the bend also varies. It can be seen from equation 6, the transverse pressure gradient also varies causing a change in intensity of the secondary flow.

4. Results and Discussion

In this section, the experimental data is analysed together with numerical predictions. The influence of the bend on the operational parameters are obtained and analysed.

4.1. *Validation of numerical results*

In section 3, we have seen that the numerical results reveal the flow patterns in the bent tube. Although these flow patterns cannot be visualized in the current experimental test rig, it will be useful to validate the numerical predictions of the pressure amplitude and the flow with the experiments, so as to gain confidence in the numerical results.

Figure 4 shows the pressure ratio at the exit of the bent tube from the measurements and the numerical predictions for several values of inlet pressure ratio.

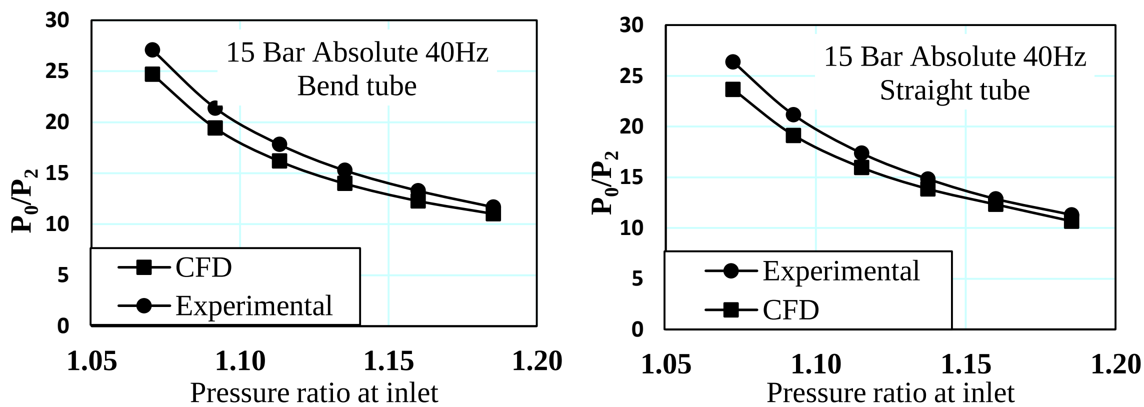

Figure 4. Comparison of non-dimensional parameter (P_0/P_2) at outlet of connecting tube.

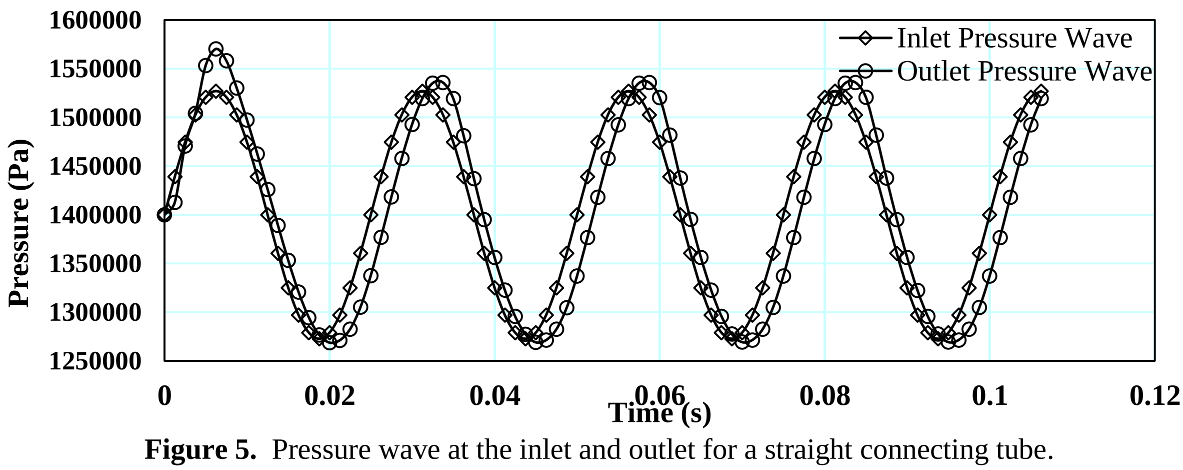

Figure 5. Pressure wave at the inlet and outlet for a straight connecting tube.

The filling pressure in this case is 1.5 MPa (P_0), P_2 is the pressure amplitude at the outlet of the connecting tube and the operating frequency is 40 Hz. In all the cases, the numerical data has a systematic difference compared to the experimental data. However, the trend of numerical data resembles the measurement data. We believe that this difference could be due to pressure drop at the

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pressure ports, which are not considered in the numerical model. The outlet pressure wave, P_2 computed by the numerical model is shown in Figure 5. The input pressure wave is the boundary condition. A phase difference of about 18 degrees can be deduced from these plots. A similar phase difference of 18 to 20 degrees is also measured for the same operating conditions for a bent tube. Therefore, the model is able to closely predict the oscillating flow characteristics in the bent tube.

4.2. Secondary flow losses and effect on volume flow

The mass flow is determined from the pressure fluctuations in the reservoir volume. The pressure amplitude at the reservoir is an indication of the mass flow rate at the connecting tube to the reservoir. Mass flow rate is calculated using the following expression

$$\dot{m} = \frac{\omega V_{res} P_1}{\gamma R T_{res}} \tag{7}$$

where ω is angular frequency, P_1 is pressure amplitude in reservoir, V_{res} is volume of the reservoir, T_{res} is the temperature of the reservoir, γ and R are the adiabatic index and characteristic gas constant of the working gas respectively. Volume flow of the gas is calculated as,

$$V = \frac{\dot{m}}{\omega \rho}. \tag{8}$$

ρ is density of working gas at charged pressure. Figure 6 shows volume flow for different frequencies.

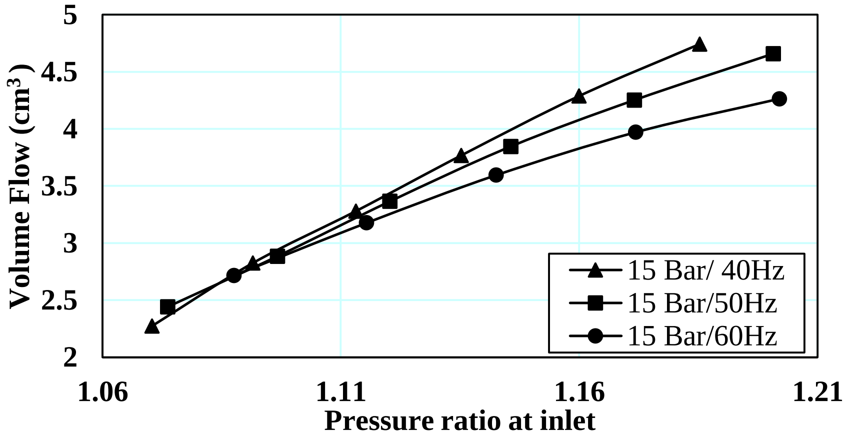

Figure 6. Volume flow for different operating frequencies.

When equation 7 is substituted into equation 8, we notice that the volume flow is independent of frequency. However, change in frequency causes change in the pressure amplitude P_1. Reduction of pressure amplitude in the reservoir results in decrease of volume flow. This reduction in pressure amplitude is because of reduced displacement of the gas at higher frequency for the same pressure ratio at the inlet. From figure 6, it is observed that the volume flow is inversely proportional to frequency. The pressure amplitude P_1 is found to be identical for different frequencies at 1.09 inlet pressure ratio. This can be due to low flow rate at low pressure ratio and hence less losses. It is also because of the inaccuracy of the measuring instrument (piezo-resistive pressure sensor). In case of bent tube minor losses add to the reduction of pressure amplitude and hence the volume flow is less for a bent tube. The minor losses are due to secondary flow in the bent tube. Difference of volume flow between straight and bent tube is shown in figure 7.

Figure 7 shows that straight tube has more volume flow due to the absence of secondary flow. As discussed above, the intensity of the secondary flow depends on the volume flow. Decrease of volume flow in the straight tube is because of increased frequency. However in a bent tube, along with the increased frequency, minor losses due to secondary flow add to the reduction in volume flow. Difference in volume flow for lower frequency is higher because of the higher strength of the secondary

flow. The difference in volume flow for a frequency of 40 Hz is larger because intensity of secondary flow is higher as compared to frequencies of 50 and 60 Hz.
For a bent tube the intensity of secondary flow varies with the frequency and is shown in figure 8.

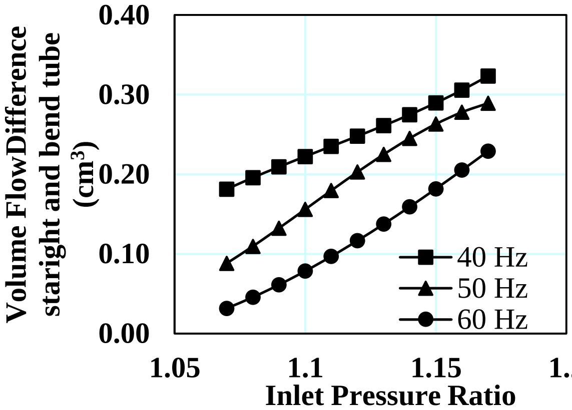

Figure 7. Difference in volume flow for straight and bent tube.

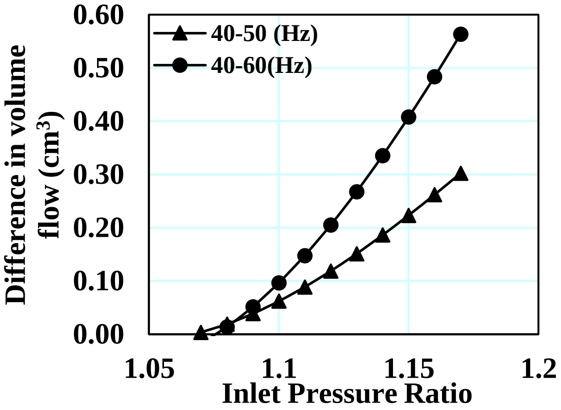

Figure 8. Strength of secondary flow in bent tube

Difference in volume flow between 40 and 60 Hz is higher than difference in volume flow between 40 and 50 Hz.This is because of two reasons. Firstly, the volume flow decreases with increase in frequency for same pressure ratio at inlet, secondly the intensity of secondary flow scales with volume flow i.e., secondary flow losses are lower for higher frequencies. The main point to focus is the variation of volume flow difference. The volume flow difference variation is approximately of the second order. This also predicts the presence of secondary flow in the bent tube and it's scaling with frequency.

Recalling equation 6, the transverse pressure gradient depends on velocity of flow (V) and density of fluid (ρ). The operating pressure effects the density of the working fluid. This leads to a change in the value of transverse pressure gradient and hence the strength of secondary flow. The velocity term involved in the equation 6 is directly proportional to the volume flow because the cross sectional area of the connecting tube is constant. Hence for identical volume flow and for a fixed radius of curvature (R), the transverse pressure gradient depends only on the density of the working fluid. Figure 9 shows the variation of dimensionless pressure ratio (P_0/P_2) for different filling pressure.

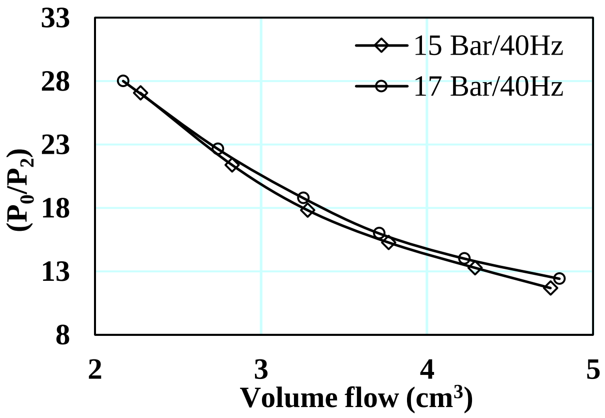

Figure 9. Variation of dimensionless pressure ratio parameter with volume flow.

From figure 9, it is observed that for the same value of volume flow, the dimensionless parameter is greater for higher charging pressure. This means the relative magnitude of P_2 is less for higher charging pressure. P_2 is the pressure amplitude at the outlet of connecting tube. Hence, at higher charged pressure i.e., higher density, there is loss of pressure amplitude at the outlet of connecting tube. This is due to increase in strength of secondary flow as seen from equation 6. As visible from the above figure, the difference in the pressure ratio (P_2/P_0) is very less, this is because the variation of density is very small between 15 and 17 bar filling pressure. But the difference is visible and hence, at higher charging pressure, the operating frequency should also be higher to reduce the volume flow as discussed earlier. This would compensate for the losses due to increase in density.

In case of U-Type Stirling pulse tube cooler, a 180 degrees bent tube connects the regenerator to the cold end heat exchanger (CHX). The acoustic power at the CHX, which is the product of pressure amplitude, volume flow amplitude and cosine of angle between volume flow and pressure amplitude, at the inlet of CHX is equal to the gross refrigeration power. The reduction in pressure amplitude due to the bend reduces the refrigeration effect. Therefore the bend should be designed to reduce secondary flow losses. Additional point of attention is that at low temperatures, the density of the working fluid increases and therefore increases the secondary flow losses. The geometry parameter, radius of the bend (R) should be optimized for a given frequency of operation and charged pressure to minimize the secondary flow losses.

5. Conclusion

Numerical and experimental methodology is adopted to characterize the pressure drop losses in a bent and a straight tube at various operating conditions. The development of secondary flow due to the curvature of the bent tube leads to pressure drop. The secondary flow intensity depend on the operating parameters, namely volume flow, frequency of operation, density of gas and also on the geometry, namely the bent radius. At lower frequency operation, the volume flow is larger for a fixed pressure ratio at the inlet, so the losses in the bend are larger. Therefore in designing transfer line at lower frequency of operation the radius of the bend should be large. At higher frequency of operation, the pressure losses in the bend are less significant. At lower temperature the density of the gas increases, therefore the bends should be gradual irrespective of the frequency of operation.

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